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Heavy tails and electricity prices

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Heavy tails and stable Paretian distributions in finance and macroeconomics

Heavy tails and electricity prices

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Abstract. In the first years after the emergence of deregulated power markets it became apparent that for the valuation of electricity derivatives we cannot simply rely on models developed for financial or other commodity markets. However, before adequate models can be put forward the unique characteristics of electricity (spot) prices have to be thoroughly analyzed. In particular, the extreme volatility and price spikes which lead to heavy-tailed distributions of returns. In this paper we first analyze the stylized facts of electricity prices, then present two modeling approaches: jump-diffusion and regime-switching, which to some extent address the pertinent issues.

1. Introduction. Since the discovery of the light bulb, electricity has made a tremendous impact on the development of our society. Today, it represents a crucial component of modern way of life, and it is hard to imagine a life without it. To provide every household with a sufficient supply of electric energy, power generator companies were set up. They used to serve dedicated geographical areas from which consumers had to buy their electricity. However, since the late 1980's dramatic changes to the structure of the electricity business have taken place around the world. The original monopolistic situation was replaced by deregulated markets, where consumers in principle were free to choose their provider – the market place for electric power had become competitive. To facilitate trading in these new free markets, exchanges for electric power have been organized. Everything from spot contracts to derivatives, like (standardized, but not marked to market) forward, futures and option contracts, are traded. Bilateral trading has evolved even more dramatically. Apart from spot and forward contracting, large numbers of structured and exotic products are used.

With deregulation and introduction of competition a new challenge has emerged for power market participants. Extreme price volatility, which can be even two orders of magnitude higher than for other commodities or financial instruments, has forced producers and wholesale consumers to hedge not only against volume risk but also against price movements. Price forecasts have become a fundamental input to an energy company's decision-making and strategy development. This in turn has propelled research in electricity price modeling and forecasting.

Electricity price modeling and forecasting can be classified in terms of the planning horizon's duration, as short-term (STPF), medium-term (MTPF) and long-term price forecasting (LTPF). However, there is no consensus as to what the thresholds should actually be. The main objective of LTPF is investment profitability analysis and planning, such as determining the future sites or fuel sources of power plants. Lead times are typically measured in years. Medium-term or monthly time horizons are generally preferred for balance sheet calculations, risk management and derivative pricing. In many cases not the actual point forecasts but the distributions of future prices over certain time periods are evaluated. As this type of modeling has a long-dated tradition in finance, inflow of "finance solutions" is readily observed; for recent reviews see Bunn and Karakatsani (2003), Eydeland and Wolyniec (2003) and Weron (2005b).

Not only monthly or annual time horizons are interesting for generators, utilities and power marketers. When bidding for spot electricity in a power exchange or a pool-type market, actors are requested to express their bids in terms of prices and quantities. Buy (sell) orders are accepted in order of increasing (decreasing) prices until total demand (supply) is met. A power plant that is able to forecast spot prices can adjust its own production schedule accordingly and hence maximize its profits. Since the day-ahead spot market typically consists of 24 hourly auctions that take place simultaneously one day in advance, forecasting with lead times from a few hours to a few days is of prime importance in day-to-day market operations (Bunn, 2000, Misiorek et al., 2006).

In this paper, however, we do not aim at reviewing the whole spectrum of electricity price modeling and forecasting. Rather we want to concentrate on medium-term modeling of spot prices with the objective of derivatives valuation. Consequently, we do not require our models to accurately forecast hourly prices but to recover the main characteristics of electricity prices at a daily time scale. In particular, the extreme volatility and price spikes which lead to heavy-tailed distributions of returns.

The paper is structured as follows. In Section 2 we will review the stylized facts of electricity prices. Naturally, we will place special emphasis on their heavy-tailed nature. In particular, in Section 3 we will analyze the distributions of electricity prices. These two sections will give us a feeling of what are the essential properties of power markets in general (and spot prices in particular) and thus give us sufficient grounds for discussing modeling approaches in Section 4. Finally, in Section 5 we will conclude and comment on derivatives pricing.

2. Stylized facts.

In this section we will review the so-called *stylized facts* of selected power markets. We will illustrate our findings mostly on data from Scandinavia, which is well known for the world's most mature power exchange Nord Pool. It also offers vast amounts of reliable and "homogeneous" data, see Fig. 1 where over 12 years of spot price data – containing 111000 observations in total – is displayed.

Many of the presented characteristics are universal, in the sense that they are shared by most electricity spot markets in the world. Yet, a few are very specific to Scandinavia. Moreover, as will be seen below, some of the features are dramatically different from those found in the financial or other commodity markets.

2.1. Price spikes. One of the most pronounced features of electricity markets are the abrupt and generally unanticipated extreme changes in the spot prices known as *jumps* or *spikes*. Within a very short period of time, the system price can increase substantially

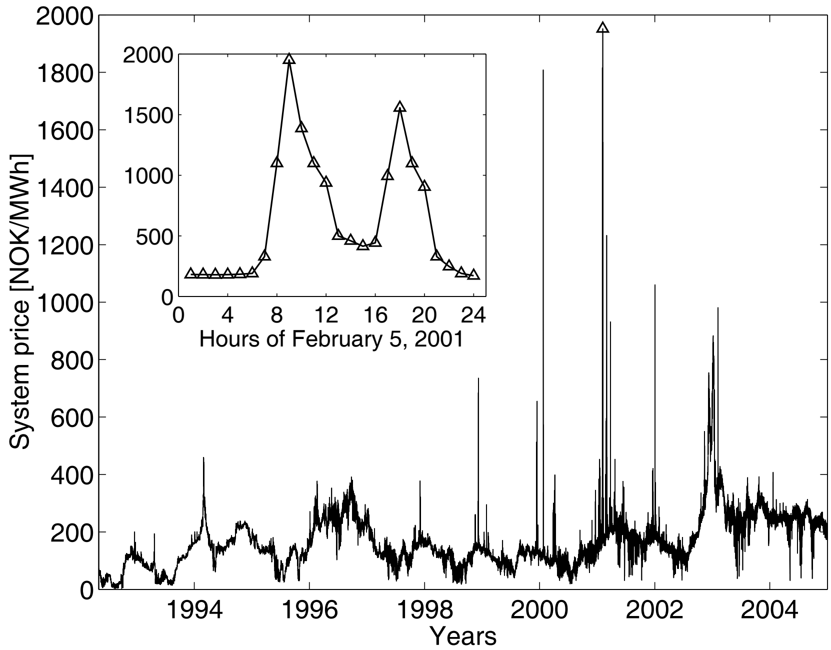

Figure 1: Hourly system price for the spot market (Elspot) at the Nordic power exchange Nord Pool from May 4, 1992 until December 31, 2004 (over 12 years of data and 111000 observations in total). In a matter of hours the price can increase tenfold leading to a price spike like that of February 5, 2001 when the price reached the all-time-high of 1951.76 NOK/MWh, see the inset.

and then drop back to the previous level, see Fig. 1 where the Nord Pool system (spot) price is depicted at an hourly time resolution.

These temporary price escalations account for a large part of the total variation of changes in spot prices and firms that are not prepared to manage the risk arising from price spikes can see their earnings for the whole year evaporate in a few hours. And we have to stress that the price of electricity is far more volatile than that of other commodities normally noted for extreme volatility. Applying the classical notion of volatility – the standard deviation of returns – we obtain that measured on the daily scale (i.e. for daily prices):

- treasury bills and notes have a volatility of less than 0.5%,
- stock indices have a moderate volatility of about 1-1.5%,
- commodities like crude oil or natural gas have volatilities of 1.5-4%,
- very volatile stocks have volatilities not exceeding 4%,
- and electricity exhibits extreme volatility – up to 50%!

The spike intensity is also non-homogeneous in time. The spikes are especially notorious during *on-peak hours*, i.e. around 9h and 18h on business days, and during high consumption periods: winter in Scandinavia, summer in mid-western U.S., etc. As the time horizon increases and the data are aggregated the spikes are less and less apparent. For weekly or monthly averages, the effects of price spikes are usually neutralized in the data.

It is not uncommon that prices from one day to the next or even within just a few hours can increase tenfold. The "spiky" nature of spot prices is the effect of non-storability of electricity. Electricity to be delivered at a specific hour cannot be substituted for electricity available shortly after or before. As currently there is no efficient technology (at a reasonable price) for storing vast amounts of power, it has to be consumed at the same time as it is produced. Hence, extreme load fluctuations – caused by severe weather conditions often in combination with generation outages or transmission failures – can lead to price spikes.

The spikes are normally quite short-lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level (Kaminski, 1999, Weron et al., 2004). For instance, on Monday, February 5, 2001, the spot price for delivery of electricity between 6 and 7h was 190.33 NOK/MWh, see the inset in Fig. 1. Three hours later, it reached the all-time-high of 1951.76 NOK/MWh, an increase of more than a factor of ten. While at the end of the day electricity was again priced moderately below 200 NOK/MWh. It should be mentioned, though, that Nord Pool is known for having less pronounced spikes than many other markets.

Despite their rarity, price spikes are the very motive for designing insurance protection against electricity price movements. This is one of the most serious reasons for including discontinuous components in realistic models of electricity price dynamics. Failing to do so, will greatly underestimate, say, the option premium, and thus increase the risk for the writer of the option. For instance in the U.S., where the size of the spikes can be much more severe, there are examples of power companies having to file for bankruptcy after having underestimated the risks related to price spikes. A textbook example is the bankruptcy of the Power Company of America (PCA), a well established power-trading company (Weron, 2005b).

2.2. Seasonality. It is well known that electricity demand exhibits seasonal fluctuations (Eydeland and Wolyniec, 2003, Kaminski, 1999, Pilipovic, 1998). They mostly arise due to changing climate conditions, like temperature and the number of daylight hours. In some countries also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. These seasonal fluctuations in demand and supply translate into seasonal behavior of electricity prices, and spot prices in particular. A typical, for the Nordic countries, behavior of the price process is presented in Fig. 2. Superimposed on the daily average system price from the Nord Pool market is a sinusoid with a linear trend. The sinusoid nearly duplicates the long-term annual fluctuations – high prices in winter time and low prices during the summers. Pilipovic (1998) and Roncoroni and Geman (2003) successfully applied the "sinusoidal" approach to a number of electricity price processes. In some markets, however, no clear annual seasonality is present and the spot prices behave similarly throughout the year with spikes occurring in all seasons.

Depending on the time resolution studied, modeling of the weekly or even the daily periodicity may be required. Apart from the annual "sinusoidal" behavior there is a

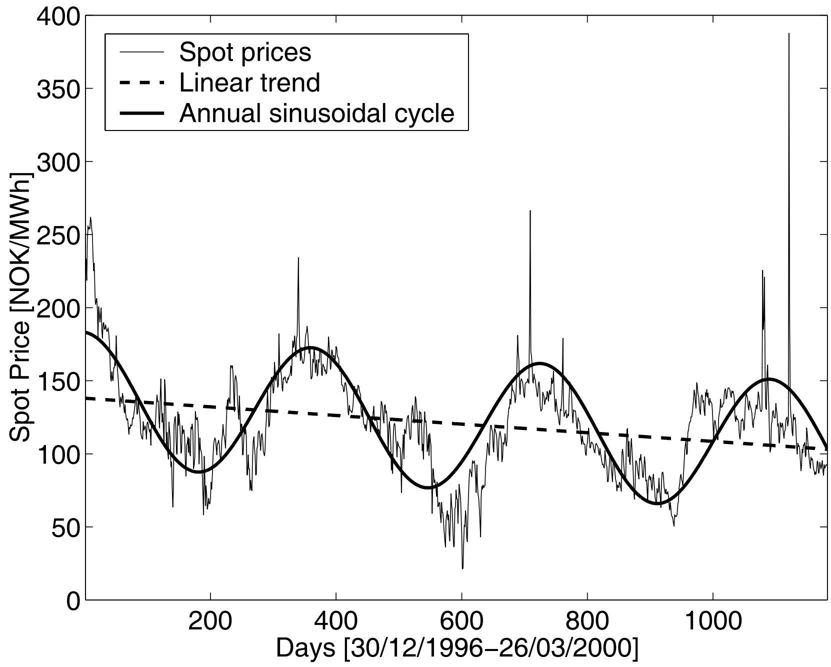

Figure 2: Nord Pool market daily average system price since December 30, 1996 until March 26, 2000 (1170 daily observations, 169 full weeks). Superimposed on the plot is an approximation of the annual seasonality by a sinusoid with a linear trend.

substantial intra-day variability. Higher than average prices are observed during the morning and evening peaks, while mid-day and night prices tend to be lower than average. The intra-week variability, related to the business day-weekend structure, is also non-negligible. The weekday prices are higher than those during the weekends, when major businesses are closed.

The modeling of intra-week and intra-day seasonalities may be approached analogously to modeling annual fluctuations, i.e. by simply taking a sine function of a one week period (Borovkova and Permana, 2004), or better a sum of sine functions with distinct periods to recover the non-sinusoidal weekly structure (Cartea and Figueroa, 2005). Alternatively, we may apply the moving average technique, which reduces to calculating the average weekly price profile (Brockwell and Davis, 1996) or just extract the mean or median week. Yet another approach was taken by Burger et al. (2004) who incorporated a SARIMA forecast of the system load into the spot price formula. In this way the seasonalities present in load data get automatically transferred to the price series. The technique is justified by the fact that the spot price is heavily dependent on the system load as a result of the supply stack structure (Weron, 2005b).

2.3. Mean Reversion. Energy spot prices are in general regarded to be mean reverting or anti-persistent. The speed of mean reversion, however, depends on several factors including the commodity being analyzed and the delivery provisions associated with the commodity (Pindyck, 1999). In electricity markets, it is common to observe sudden price spikes with very fast mean reversion to the previous price levels. In natural gas markets, the mean reversion rate is considerably slower, but the volatilities for longer-dated contracts are usually lower than the volatilities for the shorter-dated ones. In oil markets, the mean reversion rate is thought to be longer term, and it can take months, or even years, for prices to revert to their mean.

Among all financial time series spot electricity prices are perhaps the best example of anti-persistent data. Simonsen (2003) and Weron (2002) used Hurst R/S analysis, Detrended Fluctuation Analysis (DFA), Average Wavelet Coefficient (AWC) and periodogram regression methods to verify this claim for various power markets. For time intervals ranging from a day to almost four years the Hurst exponent H was found to be significantly lower than 0.5, indicating mean reversion. Importantly, these results are not an artifact of the seasonality nor the spiky character of electricity spot prices. Although, the Hurst exponent generally slightly increases after removal of seasonality and/or spikes, it still is significantly lower than 0.5. For time intervals of less than 24 hours, however, H is above 0.5, suggesting persistence on the intra-daily level.

3. Distributions of electricity prices.

It has been long known that financial asset returns are not normally distributed. Rather, the empirical observations exhibit excess kurtosis. This heavy-tailed or leptokurtic character of the distribution of price changes has been repeatedly observed in various financial and commodity markets (Carr et al., 2002, Rachev and Mittnik, 2000). The pertinent question is whether electricity prices are also heavy-tailed, and if yes, what probability distributions best describe the data.

Although the answer to the first part of the question is pretty straightforward, the second part requires further analysis. In this section we will model electricity prices with distributions from two popular heavy-tailed families (α-stable and generalized hyperbolic) and assess their goodness-of-fit.

3.1. Stable distributions. In response to the empirical evidence Mandelbrot (1963) and Fama (1965) proposed the stable distribution as an alternative model to the Gaussian law. There are at least two good reasons for modeling financial variables using stable distributions. Firstly, they are supported by the generalized Central Limit Theorem, which states that stable laws are the only possible limit distributions for properly normalized and centered sums of independent, identically distributed random variables. Secondly, stable distributions are leptokurtic. Since they can accommodate the fat tails and asymmetry, they fit empirical distributions much better.

Stable laws – also called α-stable, stable Paretian or Lévy stable – were introduced by Paul Lévy during his investigations of the behavior of sums of independent random variables in the early 20th century. The α-stable distribution requires four parameters for complete description: $\alpha \in (0, 2]$, $\beta \in [-1, 1]$, $\sigma > 0$ and $\mu \in \mathbb{R}$. The tail exponent α determines the rate at which the tails of the distribution taper off. When $\alpha = 2$, the Gaussian distribution results. When $\alpha < 2$, the variance is infinite and the tails are asymptotically equivalent to a Pareto law, i.e. they exhibit a power-law behavior. More precisely, using a central limit theorem type argument it can be shown that (Janicki and

Weron, 1994, Samorodnitsky and Taqqu, 1994):

$$\begin{cases} \lim_{x\to\infty} x^\alpha \mathbb{P}(X > x) = C_\alpha(1+\beta)\sigma^\alpha, \\ \lim_{x\to\infty} x^\alpha \mathbb{P}(X < -x) = C_\alpha(1+\beta)\sigma^\alpha, \end{cases} \tag{1}$$

where C_α is a function of α only. When $\alpha > 1$, the mean of the distribution exists and is equal to μ. When the skewness parameter β is positive (negative), the distribution is skewed to the right (left), i.e. the right (left) tail is thicker. The last two parameters, σ and μ, are the usual scale and location parameters.

From a practitioner's point of view the crucial drawback of the stable distribution is that, with the exception of three special cases ($\alpha = 2, 1, 0.5$), its probability density function (PDF) and cumulative distribution function (CDF) do not have closed form expressions. Hence, the α-stable distribution can be most conveniently described by its characteristic function $\phi(t)$ – the inverse Fourier transform of the PDF. However, there are multiple parameterizations for α-stable laws and much confusion has been caused by these different representations. The most popular parameterization of the characteristic function of $X \sim S_\alpha(\sigma, \beta, \mu)$, i.e. an α-stable random variable with parameters α, σ, β and μ, is given by (Samorodnitsky and Taqqu, 1994, Weron, 1996):

$$\log \phi(t) = \begin{cases} -\sigma^\alpha |t|^\alpha \{1 - i\beta \mathrm{sign}(t) \tan \frac{\pi\alpha}{2}\} + i\mu t, & \alpha \neq 1, \\ \\ -\sigma |t| \{1 + i\beta \mathrm{sign}(t) \frac{2}{\pi} \log |t|\} + i\mu t, & \alpha = 1. \end{cases} \tag{2}$$

For numerical purposes, it is often useful to use Nolan's (1997) parameterization:

$$\log \phi_0(t) = \begin{cases} -\sigma^\alpha |t|^\alpha \{1 + i\beta \mathrm{sign}(t) \tan \frac{\pi\alpha}{2} [(\sigma|t|)^{1-\alpha} - 1]\} + i\mu_0 t, & \alpha \neq 1, \\ \\ -\sigma |t| \{1 + i\beta \mathrm{sign}(t) \frac{2}{\pi} \log(\sigma|t|)\} + i\mu_0 t, & \alpha = 1. \end{cases} \tag{3}$$

The $S^0_\alpha(\sigma, \beta, \mu_0)$ representation is a variant of Zolotarev's (1986) (M)-parameterization, with the characteristic function and hence the density and the distribution function jointly continuous in all four parameters. In particular, percentiles and convergence to the power-law tail vary in a continuous way as α and β vary. The location parameters of the two representations are related by $\mu = \mu_0 - \beta\sigma \tan \frac{\pi\alpha}{2}$ for $\alpha \neq 1$ and $\mu = \mu_0 - \beta\sigma \frac{2}{\pi} \log \sigma$ for $\alpha = 1$.

The estimation of stable law parameters is in general severely hampered by the lack of known closed–form density functions for all but a few members of the stable family. Numerical approximation or direct numerical integration are nontrivial and burdensome from a computational point of view. As a consequence, the maximum likelihood (ML) estimation algorithm based on such approximations is difficult to implement and time consuming for samples encountered in practice. Yet, the ML estimates (Mittnik et al., 1999, Nolan, 2001) are almost always the most accurate, closely followed by the regression-type estimates (Kogon and Williams, 1998, Koutrouvelis, 1980) and McCulloch's (1986) quantile method.

Simulation of stable variates is relatively easy and involves trigonometric transformations of two independent uniform variates (Chambers et al., 1976, Weron, 1996). Other methods that utilize series representations have also been proposed but are not that universal and, in general, more computationally demanding (see Weron, 2004).

3.2. Hyperbolic distributions. In response to remarkable regularities discovered by geomorphologists in the 1940s, Barndorff-Nielsen (1977) introduced the hyperbolic law for modeling the grain size distribution of windblown sand. Excellent fits were also obtained for the log-size distribution of diamonds from a large mining area in South West Africa. Almost twenty years later the hyperbolic law was found to provide a very good model for the distributions of daily stock returns from a number of leading German enterprises (Eberlein and Keller, 1995, Küchler et al., 1999), giving way to its today's use in stock price modeling and market risk measurement. The name of the distribution is derived from the fact that its log-density forms a hyperbola. Recall that the log-density of the normal distribution is a parabola. Hence the hyperbolic distribution provides the possibility of modeling heavier tails.

The hyperbolic distribution is defined as a normal variance-mean mixture where the mixing distribution is the generalized inverse Gaussian (GIG) law with parameter $\lambda = 1$, i.e. it is conditionally Gaussian. More precisely, a random variable Z has the hyperbolic distribution if:

$$(Z|Y) \sim \mathrm{N}\left(\mu + \beta Y, Y\right), \tag{4}$$

where Y is a generalized inverse Gaussian GIG$(\lambda = 1, \chi, \psi)$ random variable and $\mathrm{N}(m, s^2)$ denotes the Gaussian distribution with mean m and variance s^2. Relation (4) implies that a hyperbolic random variable $Z \sim \mathrm{H}(\psi, \beta, \chi, \mu)$ can be represented in the form:

$$Z \sim \mu + \beta Y + \sqrt{Y}\mathrm{N}(0, 1), \tag{5}$$

with the characteristic function:

$$\phi_Z(u) = e^{iu\mu} \int_0^\infty e^{i\beta z u - \frac{1}{2} z u^2} dF_Y(z). \tag{6}$$

Here $F_Y(z)$ denotes the distribution function of a generalized inverse Gaussian random variable Y with parameter $\lambda = 1$. Hence, the hyperbolic PDF is given by:

$$f_{\mathrm{H}}(x) = \frac{\sqrt{\psi/\chi}}{2\sqrt{\psi + \beta^2} K_1(\sqrt{\psi\chi})} e^{-\sqrt{\{\psi+\beta^2\}\{\chi+(x-\mu)^2\}}+\beta(x-\mu)}, \tag{7}$$

where the normalizing constant $\mathrm{K}_\lambda(t)$ is the modified Bessel function of the third kind with index λ (here $\lambda = 1$), also known as the MacDonald function.

Sometimes another parameterization of the hyperbolic distribution with $\delta = \sqrt{\chi}$ and $\alpha = \sqrt{\psi + \beta^2}$ is used. Then the probability density function of the hyperbolic $\mathrm{H}(\alpha, \beta, \delta, \mu)$ law can be written as:

$$f_{\mathrm{H}}(x) = \frac{\sqrt{\alpha^2 - \beta^2}}{2\alpha\delta K_1(\delta\sqrt{\alpha^2 - \beta^2})} e^{-\alpha\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)}, \tag{8}$$

where $\delta > 0$ is the scale parameter, $\mu \in R$ is the location parameter and $0 \leq |\beta| < \alpha$. The latter two parameters – α and β – determine the shape, with α being responsible for the steepness and β for the skewness. The calculation of the PDF is straightforward, however, the CDF has to be numerically integrated from eqn. (8).

The hyperbolic law is a member of a more general class of generalized hyperbolic distributions, which also includes the normal-inverse Gaussian (NIG) and variance-gamma distributions as special cases. The generalized hyperbolic law can be represented as a normal variance-mean mixture where the mixing distribution is the generalized inverse Gaussian (GIG) law with any $\lambda \in \mathbb{R}$. The normal-inverse Gaussian (NIG) distributions were introduced by Barndorff-Nielsen (1995) as a subclass of the generalized hyperbolic laws obtained for $\lambda = -\frac{1}{2}$. The density of the normal-inverse Gaussian distribution is given by:

$$f_{\mathrm{NIG}}(x) = \frac{\alpha\delta}{\pi} e^{\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)} \frac{\mathrm{K}_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\sqrt{\delta^2+(x-\mu)^2}}. \tag{9}$$

Like for the hyperbolic distribution the calculation of the PDF is straightforward, but the CDF has to be numerically integrated from eqn. (9).

At the "expense" of four parameters, the NIG distribution is able to model symmetric and asymmetric distributions with possibly long tails in both directions. Its tail behavior is often classified as "semi-heavy", i.e. the tails are lighter than those of non-Gaussian stable laws, but much heavier than Gaussian. Interestingly, if we let α tend to zero the NIG distribution converges to the Cauchy distribution (with location parameter μ and scale parameter δ), which exhibits extremely heavy tails. The tail behavior of the NIG density is characterized by the following asymptotic relation:

$$f_{NIG}(x) \approx |x|^{-3/2} e^{(\mp\alpha+\beta)x} \quad \text{for} \quad x \to \pm\infty. \tag{10}$$

In fact, this is a special case of a more general relation with the exponent of $|x|$ being equal to $\lambda - 1$ (instead of $-3/2$), which is valid for all generalized hyperbolic laws (Barndorff-Nielsen and Blaesild, 1981). Obviously, the NIG distribution may not be adequate to deal with cases of extremely heavy tails such as those of Pareto or non-Gaussian stable laws. However, empirical experience suggests an excellent fit of the NIG law to financial data (Weron, 2004). Moreover, the class of normal-inverse Gaussian distributions possesses an appealing feature that the class of hyperbolic laws does not have. Namely, it is closed under convolution, i.e. a sum of two independent NIG random variables is again NIG (Barndorff-Nielsen, 1995).

The parameter estimation of generalized hyperbolic distributions can be performed by the maximum likelihood method, since there exist closed-form formulas (although, involving special functions) for the densities of these laws. The computational burden is not as heavy as for α-stable laws, but it still is considerable. The main factor for the speed of the estimation is the number of modified Bessel functions to compute. For a data set with n independent observations we need to evaluate n and $n+1$ Bessel functions for NIG and generalized hyperbolic distributions, respectively, whereas only one for the hyperbolic. This leads to a considerable reduction in the time necessary to calculate the likelihood function in the hyperbolic case. We also have to say that the optimization is challenging. Some of the parameters are hard to separate since a flat-tailed generalized hyperbolic distribution with a large scale parameter is hard to distinguish from a fat-tailed distribution with a small scale parameter, see Barndorff-Nielsen and Blaesild (1981) who observed such a behavior already for the hyperbolic law. The likelihood function with respect to these parameters then becomes very flat, and may have local mimima.

The most natural way of simulating generalized hyperbolic variables stems from the fact that they can be represented as normal variance-mean mixtures. The algorithm, based on relation (5), is fast and efficient if we have a handy way of simulating generalized inverse Gaussian variates. This is true for $\lambda = -\frac{1}{2}$. Other members of the generalized hyperbolic family are computationally more demanding (for a review see Weron, 2004).

3.3. Case Study: Distribution of EEX electricity prices. Let us look at mean daily spot (base-load) prices from the German power exchange EEX since January 1, 2001 until December 31, 2003. The prices, their first differences and the returns (i.e. first differences of the log-prices) are depicted in Fig. 3. Neither the Gaussian, nor the heavy-tailed alternatives yield a reasonable fit. The reason for this is the spurious skewness due to weekly seasonality.

If the data is filtered (deseasonalized with respect to the weekly period; the annual seasonality is not that apparent in German electricity prices) then the distribution of first differences or returns is more prone to modeling. In this sample the periodicity was removed by applying the moving average technique, which reduces to calculating the weekly profile and subtracting it from the spot prices (Brockwell and Davis, 1996, Weron, 2005b). The deseasonalized price series and their first differences are plotted in Fig. 4. The heavy-tailed nature of the phenomenon is apparent. The fits of Gaussian, hyperbolic, NIG and α-stable distributions to price changes are presented in the bottom panels of Fig. 4. The parameter estimates and goodness-of-fit statistics are summarized in Table 1. The Anderson-Darling test statistics may be treated as a weighted Kolmogorov statistics which puts more weight to the differences in the tails of the distributions. Although no asymptotic results are known for α-stable or generalized hyperbolic laws, approximate critical values for these goodness-of-fit tests can be obtained via the bootstrap technique (Čižek et al., 2005). In this paper, though, we do not perform hypothesis testing and just compare the test values. Naturally, the lower the values the better the fit. Apparently, the stable distribution yields the best fit, not only visually (where it recovers the power-law tail) but also in terms of the goodness-of-fit statistics. Both the Gaussian and hyperbolic laws largely underestimate the tails of the distribution.

Very often in practical applications not the electricity prices themselves but rather their logarithms are modeled. To discover the price distributions of log-prices we repeat the analysis for the first differences of log-prices, i.e. for price returns. This time after removing seasonality we apply the log transformation before taking the differences. The deseasonalized log-price series and their first differences are plotted in Fig. 5. The fits of Gaussian, hyperbolic, NIG and α-stable distributions to price returns are presented in the bottom panels of Fig. 5, while the parameter estimates and goodness-of-fit statistics are summarized in Table 2. Again, the stable distribution yields the best fit in terms of the goodness-of-fit statistics. But visually the supremacy is not that apparent. In

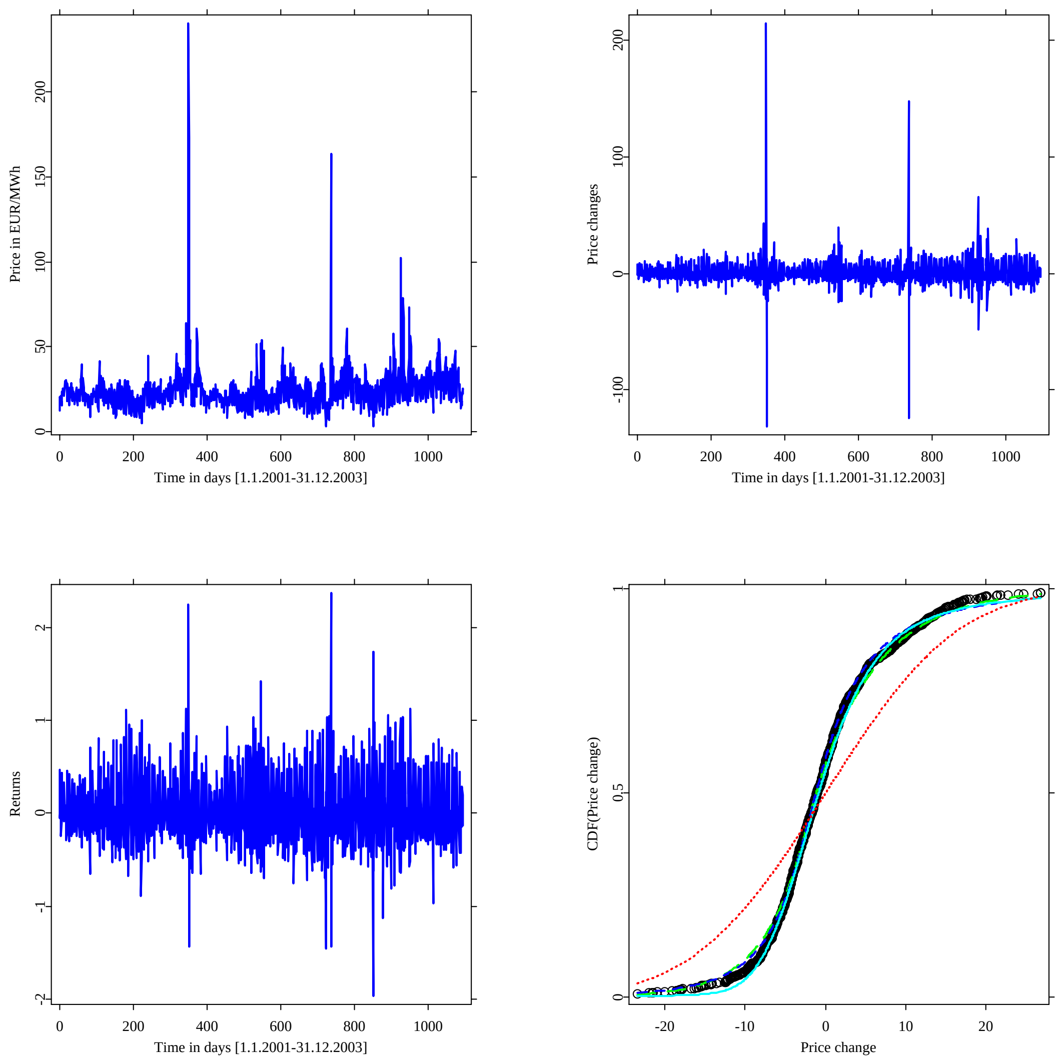

Figure 3: Mean daily EEX spot electricity prices from the period January 1, 2001 – December 31, 2003 (*top left panel*), their first differences (*top right panel*) and their returns (*bottom left panel*). *Bottom right panel:* The empirical and fitted CDFs to the price differences: Gaussian (red dotted), hyperbolic (green long-dashed), NIG (blue dashed) and α-stable (cyan solid). None of the distributions gives a reasonable fit. The reason for this is the spurious skewness due to weekly seasonality.

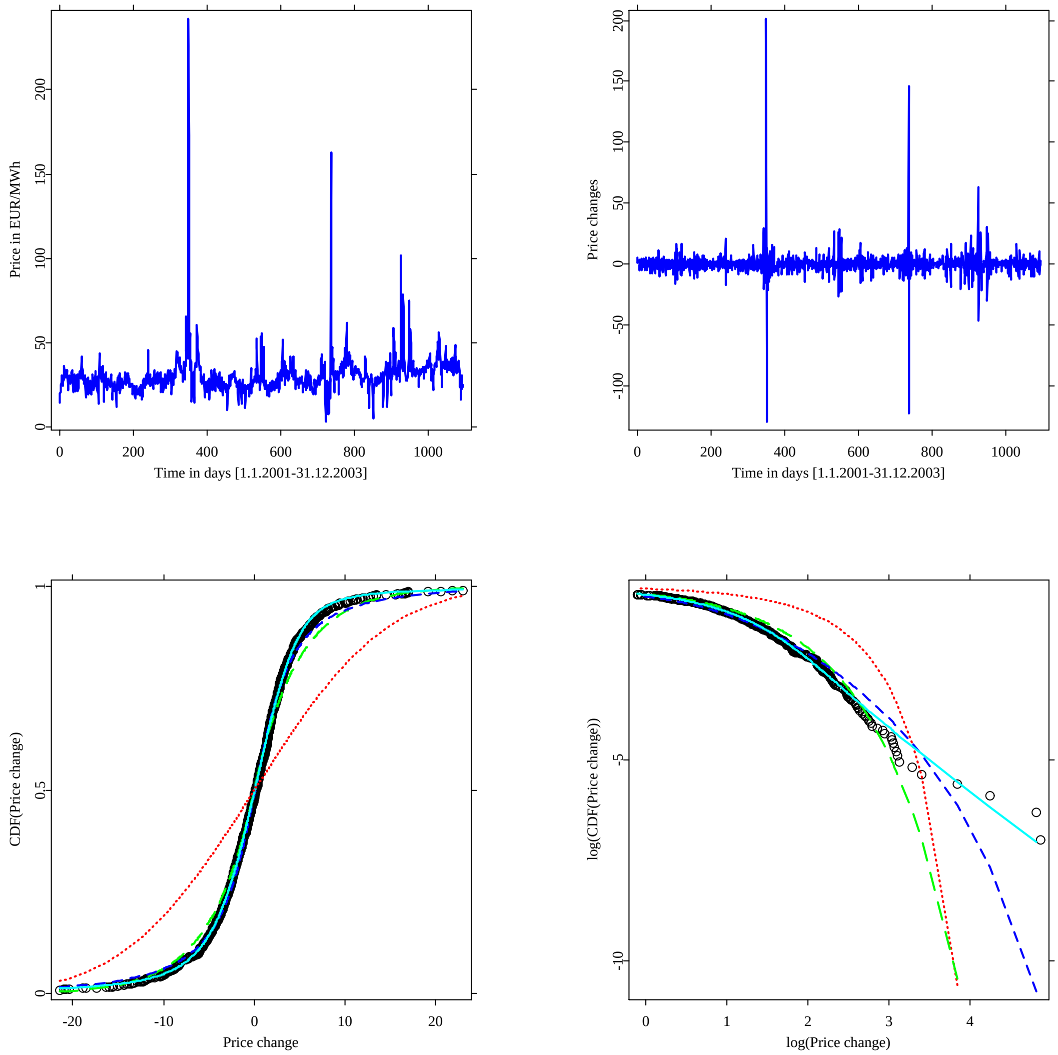

Figure 4: Deseasonalized (with respect to the weekly period) mean daily EEX spot electricity prices from the period January 1, 2001 – December 31, 2003 (*top left panel*) and their first differences (*top right panel*). *Bottom left panel:* The empirical and fitted CDFs to the price differences: Gaussian (red dotted), hyperbolic (green long-dashed), NIG (blue dashed), and α-stable (cyan solid). *Bottom right panel:* The heavy-tailed nature of the phenomenon is apparent from the double logarithmic plot of the left tail of the price distribution.

Table 1: Parameter estimates and goodness-of-fit statistics for Gaussian, hyperbolic, NIG and α-stable distributions fitted to the first differences of the deseasonalized (with respect to the weekly period) mean daily EEX spot electricity prices from the period January 1, 2001 – December 31, 2003. The symbol "+INF" denotes a very large number (infinity in computer arithmetic).

Parameters	α	σ, δ	β	μ
Gaussian fit		11.4548		0.0083
Hyperbolic fit	0.2099	0.0851	−0.0001	0.0136
NIG fit	0.0469	3.2181	−0.0031	0.0083
α-stable fit	1.5104	2.9005	−0.2616	−0.4898
Test values	**Anderson-Darling**		**Kolmogorov**	
Gaussian fit	+INF		6.9894	
Hyperbolic fit	+INF		1.8669	
NIG fit	1.7890		0.9138	
α-stable fit	0.5419		0.6831	

Table 2: Parameter estimates and goodness-of-fit statistics for Gaussian, hyperbolic, NIG and α-stable distributions fitted to the returns of the deseasonalized (with respect to the weekly period) mean daily EEX spot electricity prices from the period January 1, 2001 – December 31, 2003. The symbol "+INF" denotes a very large number (infinity in computer arithmetic).

Parameters	α	σ, δ	β	μ
Gaussian fit		24.4395		0.0445
Hyperbolic fit	0.0664	0.3653	0.0001	0.0047
NIG fit	0.0233	12.6781	−0.0014	0.0445
α-stable fit	1.4837	9.9668	−0.1915	−1.3267
Test values	**Anderson-Darling**		**Kolmogorov**	
Gaussian fit	+INF		4.0124	
Hyperbolic fit	2.6215		1.1440	
NIG fit	0.7570		0.7752	
α-stable fit	0.5237		0.6603	

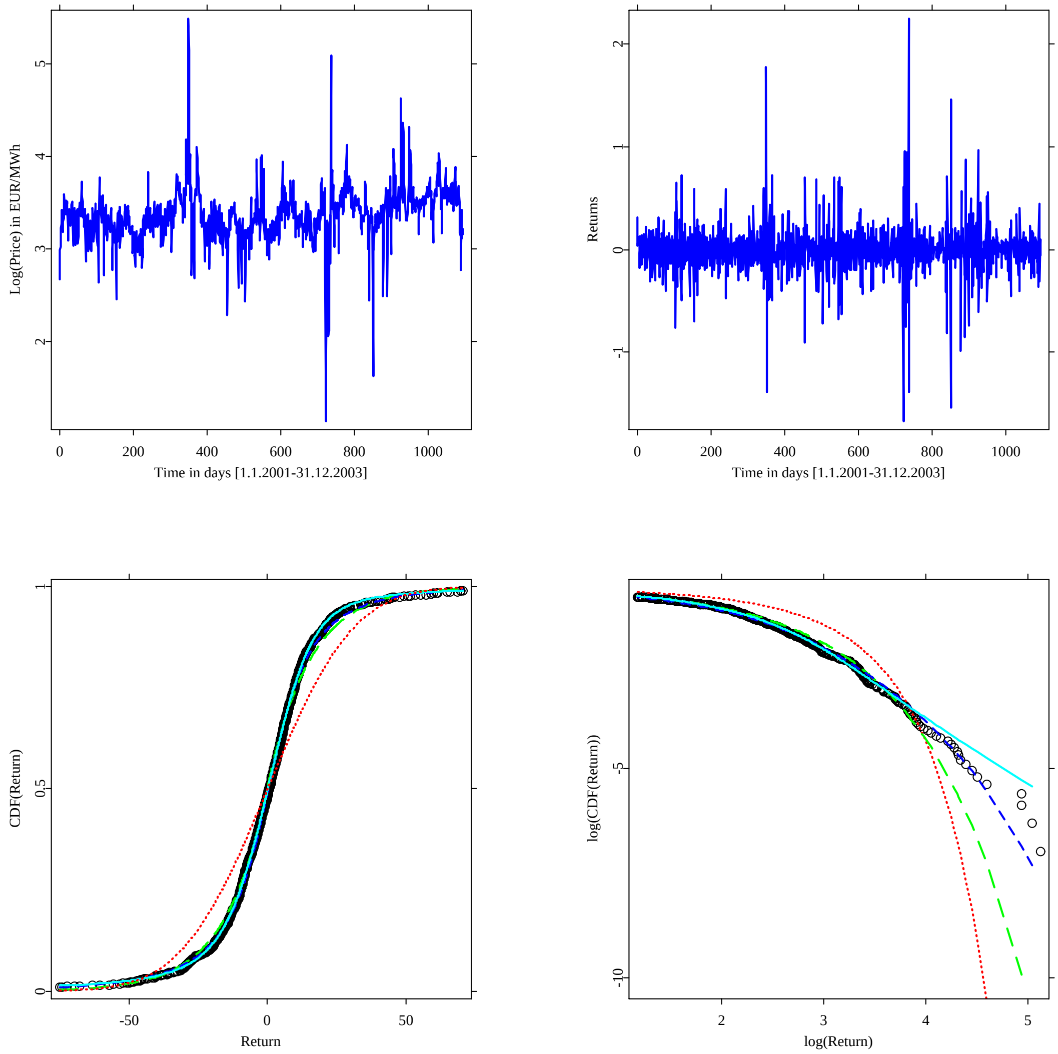

Figure 5: Logarithm of the deseasonalized (with respect to the weekly period) mean daily EEX spot electricity prices from the period January 1, 2001 – December 31, 2003 (*top left panel*) and their first differences, i.e. price returns (*top right panel*). *Bottom left panel:* The empirical and fitted CDFs to the price returns: Gaussian (red dotted), hyperbolic (green long-dashed), NIG (blue dashed), and α-stable (cyan solid). *Bottom right panel:* The heavy-tailed nature of the phenomenon is apparent, but the tails are lighter than Paretian (power-law).

fact the second in line NIG distribution also gives relatively low goodness-of-fit statistics. Like before, the Gaussian and hyperbolic laws largely underestimate the tails of the distribution.

3.4. Case Study: Distribution of Nord Pool electricity prices. Let us now calibrate these four distributions to Nord Pool market daily average system prices (denoted by P_t) from December 30, 1996 until March 26, 2000. In contrast to German spot prices, the Scandinavian data exhibits a well pronounced annual cycle, see Fig. 2. It can be quite well approximated by a sinusoid of the form:

$$S_t = A \sin\left(\frac{2\pi}{365}(t+B)\right) + Ct. \tag{11}$$

Following Weron (2005a) we propose to estimate the parameters through a two step procedure. First, a least squares fit is used to obtain initial estimates of all three parameters (A, B and C). Then the time shift parameter B is chosen such as to maximize the p-value of the Bera-Jarque test for normality applied to the deseasonalized and spikeless log-prices (for spike definition and identification procedure see Case Study 4.2) yielding: $\hat{A} = 45.19$, $\hat{B} = 94.8$ and $\hat{C} = -0.0295$.

Like in case of EEX data, we deal with the intra-week variations by preprocessing the data using the moving average technique, which reduces to calculating the weekly profile s_t and subtracting it from the spot prices. In this Case Study we fit the distributions to the logarithm of the deseasonalized prices (with respect to the weekly and annual cycles; in short: deseasonalized log-prices):

$$d_t = \log(P_t - s_t - S_t). \tag{12}$$

The time series d_t is plotted in the top panel of Fig. 6.

The fits of Gaussian, hyperbolic, NIG and α-stable distributions to price returns are presented in the bottom panels of Fig. 6, while the parameter estimates and goodness-of-fit statistics are summarized in Table 3. The stable distribution yields the best fit, both visually and in terms of the goodness-of-fit statistics. Note the extremely low value of the Anderson-Darling statistics implying a very good fit of the tails of the empirical distribution. Like for EEX data, the Gaussian and hyperbolic laws largely underestimate the tails of the distribution.

4. Modeling and forecasting electricity prices. Price process models lie at the heart of derivatives pricing and risk management systems. If the price process chosen is inappropriate to capture the main characteristics of electricity prices, the results from the model are likely to be unreliable. On the other hand, if the model is too complex the computational burden will prevent its on-line use in trading departments. Econometric models offer the best of the two worlds; they are a trade-off between model parsimony and adequacy to capture the unique characteristics of power prices.

The very good fit of the α-stable distribution to electricity price returns (documented in Case Studies 3.3-3.4; see also Mugele et al., 2005, Rachev et al., 2004), could be the motive for applying α-stable Lévy motion to modeling electricity prices. We have

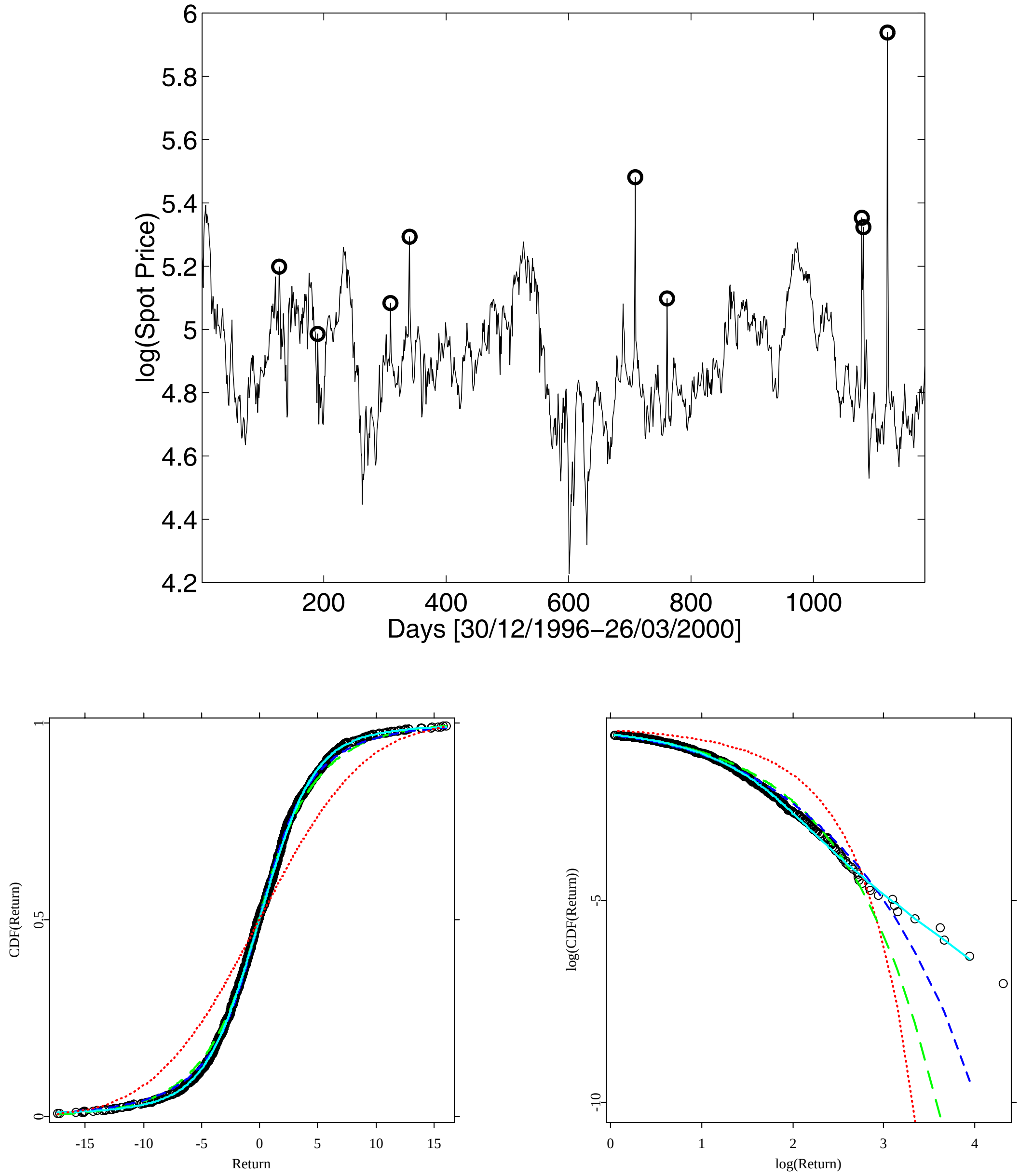

Figure 6: *Top panel:* Logarithm of the deseasonalized, with respect to the annual and weekly cycles, Nord Pool market daily average system price since December 30, 1996 until March 26, 2000 (1170 daily observations, 169 full weeks). Circles denote spikes; for spike definition see Case Study 4.2. *Bottom left panel:* The empirical and fitted CDFs to the returns of the mean daily deseasonalized (with respect to the weekly and annual periods) Nord Pool spot electricity prices from the period December 30, 1996 – March 26, 2000 (see Fig. 2): Gaussian (red dotted), hyperbolic (green long-dashed), NIG (blue dashed), and α-stable (cyan solid). *Bottom right panel:* The power-law type left tail is clearly visible in the double logarithmic plot.

Table 3: Parameter estimates and goodness-of-fit statistics for Gaussian, hyperbolic, NIG and α-stable distributions fitted to the returns of the mean daily deseasonalized (with respect to the weekly and annual periods) Nord Pool spot electricity prices from the period December 30, 1996 – March 26, 2000. The symbol "+INF" denotes a very large number (infinity in computer arithmetic).

Parameters	α	σ, δ	β	μ
Gaussian fit		7.0514		−0.0280
Hyperbolic fit	0.2587	0.8437	0.0029	−0.1179
NIG fit	0.1033	3.6808	−0.0009	−0.0280
α-stable fit	1.6078	2.8062	−0.0115	−0.0431
Test values	Anderson-Darling		Kolmogorov	
Gaussian fit	+INF		4.3882	
Hyperbolic fit	+INF		0.9933	
NIG fit	0.9621		0.7333	
α-stable fit	0.2025		0.4861	

to remember, though, that extreme price changes or returns are generally coupled in "up-jump"−"down-jump" pairs constituting the price spikes (at least at the daily time scale). Consequently, although (α-stable) Lévy motion can recover the distributional properties of returns very well it is not a good candidate for the model of electricity prices. Besides missing the "up-jump"−"down-jump" correlation it does not allow for control of the intensity of the jumps, a property that might be crucial in some power markets. In this section we discuss two alternative approaches that not only recover the distributional properties of electricity prices but also the spiky behavior.

4.1. Jump-diffusion models. As it is very natural to approach a problem by adapting already known solutions, it was only a question of time before standard stochastic models of modern finance found their way to the power market. However, the most prominent of all models – geometric Brownian motion (GBM) – could not be applied directly to electricity prices. It does not allow for price spikes and mean-reversion.

Early modeling approaches involved modifications of GBM that would allow for exactly these two electricity price characteristics. Kaminski (1997) utilized the *jump-diffusion* model of Merton (1976), which is essentially constructed by adding a Poisson (or jump) component to standard GBM. Its main drawback is that it ignores another fundamental feature of electricity prices: the mean-reversion to the "normal" price regime. If a price spike occurred GBM would "accept" the new price level as a normal event and would proceed randomly from there with no consideration of prior price levels, and a small chance of returning to the pre-spike level.

In a comparative study Johnson and Barz (1999) evaluated the effectiveness of various diffusion-type models in describing the evolution of spot electricity prices in several different markets. Apart from arithmetic and geometric Brownian motion, they tested *mean-reverting diffusion* (also known as the arithmetic Ornstein-Uhlenbeck process; originally proposed by Vasicek (1977) for modeling interest rate dynamics) and *geometric mean-reverting diffusion* with and without jumps in the form of a compound Poisson process. The authors concluded that the geometric mean-reverting jump-diffusion model gave the best performance and that all models without jumps were inappropriate for modeling electricity prices.

A general specification of jump-diffusion models involves a stochastic differential equation (SDE) that governs the dynamics of the price process:

$$dp_t = \mu(p_t, t)dt + \sigma(p_t, t)dW_t + dq(p_t, t). \tag{13}$$

The Wiener process W_t is responsible for small fluctuations (around the long-term mean for mean-reverting processes) and the pure jump process $q(p_t, t)$ produces infrequent, but large upward jumps. The latter is a compound Poisson process with given intensity and severity of jumps (see Chapter 14 in Čižek et al., 2005), typically independent of W_t. As in the models investigated by Johnson and Barz (1999), the drift term $\mu(p_t, t)$ usually forces mean-reversion to a stochastic or deterministic long-term mean at a constant rate, for instance, it could be of the form $\mu(p_t, t) = \alpha - \beta p_t = \beta(\frac{\alpha}{\beta} - p_t)$. For simplicity, the volatility term $\sigma(p_t, t)$ is often set to a constant. However, empirical evidence suggests that electricity prices exhibit heteroscedasticity (Karakatsani and Bunn, 2004, Misiorek et al., 2006) and alternative specifications have been postulated (Deng, 1999, Escribano et al., 2002).

A serious flaw of both the arithmetic and geometric mean-reverting jump-diffusion models is the slow speed of mean-reversion after a jump. When electricity prices spike, they tend to return to their mean reversion levels much faster than when they suffer smaller shocks. However, a high rate of mean-reversion β, required to force the price back to its normal level after a jump, would lead to a highly overestimated β for prices outside the "spike regime". To circumvent this, Escribano et al. (2002) allowed signed jumps. But if these randomly follow each other, the spike shape has obviously a very low probability to be generated. Roncoroni and Geman (2003) suggested using mean-reversion coupled with upward and downward jumps, with the direction of a jump being dependent on the current price level. Weron et al. (2004) postulated that a positive jump be always followed by a negative jump of (approximately) the same size to capture the rapid decline of electricity prices after a spike. On the daily level, i.e. when analyzing average daily prices, this approach seems to be a good approximation since spikes typically do not last more than a day. Borovkova and Permana (2004) proposed the drift to be given by a potential function, which forces the price to return to its seasonal level after an upward jump. Interestingly, it allows the rate of mean-reversion to be a continuous function of the distance from this level. Other modifications have been also proposed including time-varying parameters, regime-switching and stochastic volatility (Deng, 1999, Escribano et al., 2002).

Another limitation of the mean-reverting jump-diffusion model is that it assumes the (mean-reverting) diffusion process to be independent of the Poisson component. This is not the case in electricity. In particular, prices are highly unlikely to spike overnight when demand and prices are very low. To cope with this observation Eydeland and Geman (1999) proposed a model where the jump size is proportional to the current spot price. As a result the spikes tend to be more severe during high price periods.

Furthermore, empirical data suggests that the homogeneous Poisson process may not be the best choice for the jump component. Price spikes are seasonal; they typically show up in high price seasons, like winter in Scandinavia and summer in central U.S. For this reason Weron (2005a) considered using a non-homogeneous Poisson process (NHPP) with a (deterministic) periodic intensity function, instead of a HPP with a constant jump intensity rate.[1] However, the scarcity of jumps identified by the filtering procedure (only nine in over three years of Nord Pool data; see Case Study 4.2) made identification of any adequate periodic function problematic. This paucity of spikes did not refrain Roncoroni and Geman (2003) from fitting NHPPs to spike occurrences in three major U.S. power markets (COB, PJM and ECAR), despite using even shorter time series consisting of only 750 daily[2] average prices from the period 1997-1999. A highly convex, two parameter periodic intensity function was chosen to ensure that the price jump occurrence clusters around the peak dates and rapidly fades away; this effect indeed can be observed for PJM and ECAR prices, but not for COB. The parameters were identified using 6, 16 and 27 (for COB, PJM and ECAR, respectively) spike occurrences, which makes the calibration results highly questionable, especially for the COB market. Hopefully, when larger homogeneous datasets become available (or perhaps when hourly data are considered) the application of NHPPs will be statistically justified.

4.2. Case Study: A mean-reverting jump diffusion model for Nord Pool spot prices. Let us now calibrate a jump diffusion model to Nord Pool market daily average system prices (denoted by P_t) from December 30, 1996 until March 26, 2000. The weekly (s_t) and annual (S_t) seasonalities are first removed from the original prices (for details see Case Study 3.4), then a log transformation is applied. The resulting time series $d_t = \log(P_t - s_t - S_t)$ is plotted in Fig. 6.

Reflecting the fact that on the daily scale spikes typically do not last more than one time point (i.e. one day), like in Weron et al. (2004), we let a positive jump be always followed by a negative jump of about the same magnitude. This is achieved by letting d_t be the sum of a mean reverting stochastic part X_t and an independent jump component. The jump component is modeled by a compound Poisson process of the form $J_t dq_t$, where J_t is a random variable responsible for the spike severity and q_t is a (homogeneous) Poisson process with intensity κ.

The choice of J_t and κ depends on the definition of the spike. We adopt the following: a spike is an increase in the log-price (formally: an increase in d_t) exceeding $H = 2.5$ standard deviations of all price changes (i.e. $d_t - d_{t-1}$) followed by a decrease in the price. The threshold level is set arbitrarily. The usual threshold $H = 3$ results in only

[1] For a review of Poisson processes see e.g. Chapter 14 in Čižek et al. (2005).

[2] Apparently the datasets comprised only business day prices.

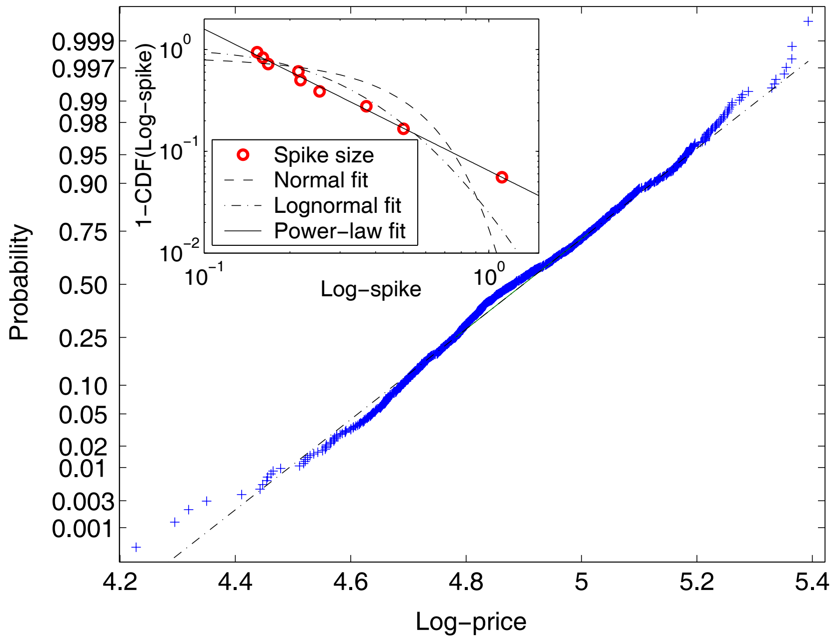

Figure 7: The normal probability plot of the stochastic part X_t of the deseasonalized log-price d_t in eqn. (14). The crosses form a straight line indicating a Gaussian distribution. The tail of the spike size distribution is depicted in the inset. Clearly, the normal and log-normal laws underestimate the severity of extreme spikes; spikes rather follow a power-law of order $x^{-1.4}$.

six spikes in the whole series, while $H = 2.5$ yields nine spikes and captures all "obvious" peaks seen in the plot of d_t, see Fig. 6. The extraction of the spikes from the original series is performed iteratively – the algorithm filters the series and removes all price changes greater than H standard deviations of all price changes at that specific iteration. The algorithm is repeated until no further spikes can be filtered. After the spikes are extracted, the price d_t at these time points is replaced by the arithmetic average of the two neighboring prices yielding the deseasonalized and "spikeless" log-prices X_t.

The extracted nine spikes do not allow for a sound statistical analysis of the spike severity nor intensity. Nevertheless, we fitted Gaussian, log-normal and Pareto loss distributions to spike sizes. The spike severities constitute a power-law of order $x^{-1.4}$ (see the inset in Fig. 7), hence, the Pareto law should yield a good fit. Unfortunately both moment and maximum likelihood estimates return unreasonable values for the parameters, either out of range or a few orders of magnitude higher than the slope of the power-law fit. The log-normal distribution $\log J_t \sim N(\mu, \rho^2)$ with $\hat{\mu} = -1.2774$ and $\hat{\rho} = 0.65124$ is the next best. Since J_t represents the size of the logarithm of the spike magnitude it is truncated at the maximum price attainable in the market (10000 NOK) to ensure a finite mean of the price process P_t. Moreover, we let q_t be a HPP with intensity $\kappa = 0.0076207$. Again the sample suggests that this may not be the best choice – six spikes were observed in winter and only one in each of the other seasons. However, rigorous estimation of a periodic intensity function (of a NHPP) using only nine time points is not possible.

Putting all the facts together, the jump diffusion model has the following form:

$$d_t = J_t dq_t + X_t \qquad \text{or} \qquad P_t = s_t + S_t + e^{J_t dq_t + X_t}, \tag{14}$$

where X_t is the stochastic component. The exponent in the last term of eqn. (14) reflects the fact that the marginal distribution of X_t is approximately Gaussian, whereas the deseasonalized, with respect to the weekly and annual cycles, and "spikeless" spot prices can be very well described by a log-normal distribution, i.e. their logarithms are approximately Gaussian. The fit is surprisingly good, the Bera-Jarque test for normality yields a p-value of 0.97, see also Fig. 7. For comparison, the p-value for the "spiky" deseasonalized log-prices d_t is less than 0.0001, allowing us to reject normality at any reasonable level. This jump diffusion model was successfully used by Weron (2005a) for pricing Asian electricity options.

4.3. Regime-switching models. The "spiky" character of spot electricity prices suggests that there exists a non-linear mechanism switching between normal and high-price states or regimes. This observation gave Robinson (2000) the grounds to fit a *logistic smooth transition autoregressive* (LSTAR) model to prices in the English and Welsh wholesale electricity Pool. He showed that the LSTAR model performed superior to a linear autoregressive alternative. However, we believe that the regime-switching mechanism is governed by an unobservable process rather than the price process itself. The spot electricity price is the outcome a vast number of variables including fundamentals (like loads and network constraints) but also the unquantifiable psycho- and sociological factors that can cause an unexpected and irrational buyout of certain commodities or contracts leading to pronounced price spikes (Misiorek et al., 2006).

In this context the *regime-switching* or *Markov-switching* models seem to be an adequate non-linear alternative. In particular, they allow for spikes that last for more than just one time period (an hour, a day), without the disadvantage of slow mean-reversion after a jump. Their usefulness has been already recognized and a number of models for spot electricity prices have been proposed (Bierbrauer et al., 2004, Ethier and Mount, 1998, Huisman and de Jong, 2003, Huisman and Mahieu, 2003, Weron et al., 2004).

The underlying idea behind the regime-switching scheme is to model the observed stochastic behavior of a specific time series by two (or more) separate phases or regimes with different underlying processes. In other words the parameters of the underlying process may change for a certain period of time and then fall back to their original structure. Thus, regime-switching models divide the time series into different phases that are called *regimes*. For each regime one can define separate and independent different underlying price processes. The switching mechanism between the states is assumed to be governed by an unobserved random variable (Franses and van Dijk, 2000). For example, the spot price can be assumed to display either low or very high volatility at each point in time, depending on the regime $R_t = 1$ or $R_t = 2$. Consequently, we have a probability law that governs the transition from one state to another. The price processes $p_{R_t,t}$ being linked to each of the two regimes are supposed to be independent from each other. The transition matrix $\mathbf{Q}$ contains the probabilities q_{ij} of switching from regime i at time t to regime j at time $t+1$, for $i, j = \{1, 2\}$:

$$\mathbf{Q} = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{pmatrix}. \tag{15}$$

Because of the Markov property the current state R_t at time t of a Markov chain depends on the past only through the most recent value R_{t-1}:

$$P\{p_t = j | p_{t-1} = i, p_{t-2} = k, \ldots\} = P\{p_t = j | p_{t-1} = i\} = p_{ij}. \tag{16}$$

Consequently the probability of being in state j at time $t+m$ starting from state i at time t is given by

$$(\mathbb{P}(R_{t+m} = j \mid R_t = i))_{i,j=1,2} = (\mathbf{Q}')^m \cdot e_i, \tag{17}$$

where $\mathbf{Q}'$ denotes the transpose of $\mathbf{Q}$ and e_i denotes the ith column of the 2×2 identity matrix.

The variety of regime-switching models is due to the possibility of choosing both the number of regimes (2, 3, etc.) and the different stochastic process for the price in each regime. Especially for the spike regime it may be interesting to choose alternative distributions. Since spikes happen very rarely but usually are of great magnitude the use of heavy-tailed distributions (like Pareto, Burr, etc.) could be considered. Also the process that switches between the states could be chosen in accordance with the typical behavior of spot electricity prices.

Parameter estimation of the processes in the regime-switching models is not straightforward since the regime is only latent and hence not directly observable. Hamilton (1990) introduced an application of the *Expectation-Maximization* (EM) algorithm of Dempster et al. (1977) where the whole set of parameters θ is estimated by an iterative two-step procedure. Based on starting values $\hat{\theta}^{(0)}$ for the parameter vector θ of the underlying stochastic processes in the first step the conditional probabilities $\mathbb{P}(R_t = j | p_1, ..., p_T; \theta)$ for the process being in regime j at time t are calculated. The probabilities are referred to as *smoothed inferences*. Then in the second step new and more exact maximum likelihood estimates $\hat{\theta}$ for all model parameters are calculated by using the smoothed inferences from step 1. With each new vector $\hat{\theta}^{(n)}$ the next cycle of the algorithm is started in order to reevaluate the smoothed inferences and so on.

Every iteration of the EM algorithm generates new estimates $\hat{\theta}^{(n+1)}$ as well as new estimates for the smoothed inferences. Hamilton (1990) shows that each iteration cycle of the sample increases the log-likelihood function and the limit of this sequence of estimates reaches a (local) maximum of the log-likelihood function.

4.4. Case Study: Regime-switching models for Nord Pool spot prices. In this Case Study we fit three regime-switching models to the logarithm of the deseasonalized average daily spot prices from the Nord Pool power exchange since January 1, 1997 until April 25, 2000. For details on obtaining d_t from raw data see Case Studies 3.4 and 4.2. The deseasonalized data exhibits several extreme events that can be considered as spikes, see Fig. 6. While most spikes only last for one day there are periods where the prices

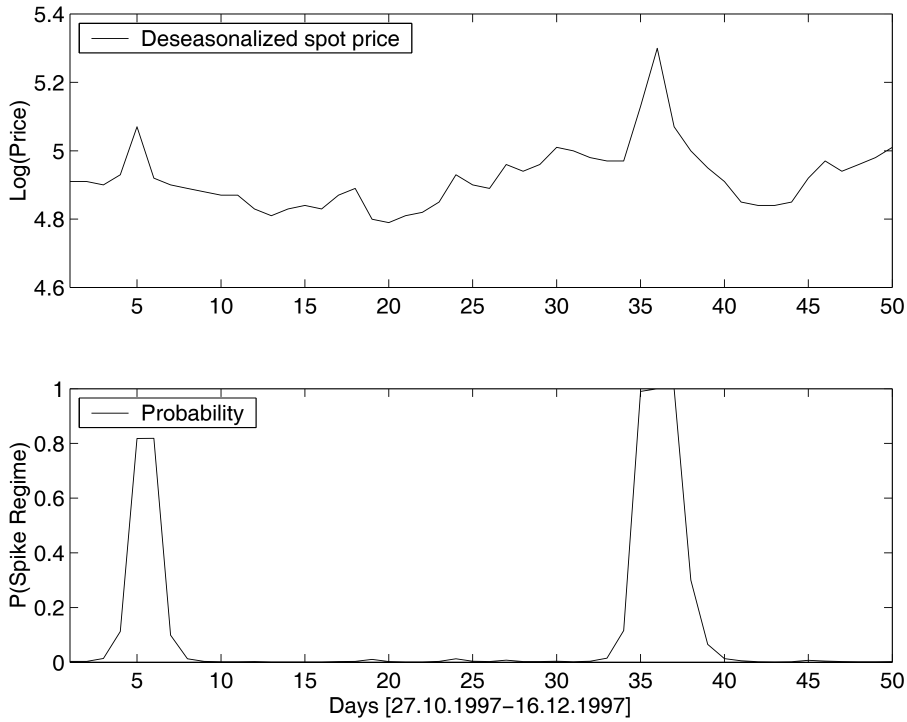

Figure 8: The deseasonalized spot Nord Pool log-price d_t since October 27, 1997 until December 16, 1997 (*top panel*). The probability of being in the spike regime for the two-regime model with log-normal spikes (*bottom panel*).

exhibit three or more extreme events in a row, a behavior that could be considered as consecutive spikes, see the top panel in Fig. 8. This is the motivation for fitting the two-regime models with the base regime dynamics given by

$$dY_{t,1} = (c_1 - \beta Y_{t,1})dt + \sigma_1 dW_t, \tag{18}$$

and the dynamics in the spike regime following three different distributions:

- Gaussian
$$Y_{t,2} \sim \mathrm{N}(c_2, \sigma_2^2),$$
- log-normal
$$\log(Y_{t,2}) \sim \mathrm{N}(c_2, \sigma_2^2),$$
- Pareto
$$Y_{t,2} \sim F_{\mathrm{Pareto}}(c_2, \sigma_2^2) = 1 - \left(\frac{c_2}{x}\right)^{\sigma_2^2}.$$

The estimation results are summarized in Table 4. As expected, in all models the probability of remaining in the base regime is very high: $q_{11} \approx 0.98$ for the Gaussian and log-normal and $q_{11} = 0.9842$ for the Pareto specification. However, the probability of remaining in the spike regime is also relatively high: $q_{22} = 0.6337$ for the Gaussian,

Table 4: Estimation results for the two-regime models fitted to the deseasonalized log-price d_t for the period January 1, 1997 – April 25, 2000. $\mathbb{E}(Y_{t,i})$ is the level of mean reversion for the base regime ($i = 1$) and the expected value of the spike regime ($i = 2$), q_{ii} is the probability of remaining in the same regime in the next time step and $\mathbb{P}(R = i)$ is the unconditional probability of being in regime i.

Regime	Parameter Estimates			Statistics			
	β	c_i	σ_i^2	$\mathbb{E}(Y_{t,i})$	$\mathbb{V}\mathrm{ar}(Y_{t,i})$	p_{ii}	$\mathbb{P}(R = i)$
Two-regime model with Gaussian spikes							
Base	0.0427	0.2086	0.0018	4.8801	0.0216	0.9802	0.9489
Spike	—	4.9704	0.0610	4.9704	0.0610	0.6337	0.0511
Two-regime model with lognormal spikes							
Base	0.0426	0.2078	0.0018	4.8807	0.0217	0.9800	0.9485
Spike	—	1.6018	0.0024	4.9678	0.0600	0.6325	0.0515
Two-regime model with Pareto spikes							
Base	0.0427	0.2087	0.0020	4.8837	0.0231	0.9842	0.9664
Spike	—	6.6848	4.2382	4.9837	0.7931	0.5464	0.0336

$q_{22} = 0.6325$ for the log-normal and $q_{22} = 0.5464$ for the Pareto model. The data points with a high probability of being in the jump regime, $\mathbb{P}(R_t = 2) > 0.5$, tend to be grouped in blocks, see Fig. 8.

Considering the unconditional probabilities we find that there is a 5.11%, 5.15% and 3.36% probability of being in the spike regime for the Gaussian, log-normal and Pareto two-regime models, respectively. Surprisingly, the Gaussian and log-normal distributions produce almost identical results. A closer inspection of the parameter estimates uncovers the mystery – with such a choice of parameter values the log-normal distribution very much resembles the Gaussian law. However, using a heavy-tailed distribution, like the Pareto law, gives lower probabilities for being and remaining in the spike regime and a clearly higher variance.

Simulated price trajectories were used to check for similarity with real prices and stability of results. Reestimating the models with simulated data led to only slightly biased estimates for the parameters. We also checked the simulation results considering spikes as the most distinguished feature of electricity spot prices, see Table 5. Defining a spike as a change in the log-prices that is greater than 30% – either in positive or negative direction – we find that the regime-switching models produce significantly more spikes than there could be observed in real data. While the number of extreme events are overestimated in all models (see the values of the upper quantiles $v_{0.99}$ and $v_{0.995}$ in Table 5), the magnitude of the largest spike in either direction is underestimated in

Table 5: Performance of the estimated regime-switching models is assessed by comparing the number of spikes, the return distributions' upper quantiles ($v_{0.99}$ and $v_{0.995}$) and the extreme events.

	# spikes	$v_{0.99}$	$v_{0.995}$	max	min
Data (d_t)	9.00	0.1628	0.2235	1.1167	−0.7469
2-regime (normal)	17.26	0.3310	0.4523	0.7580	−0.8038
2-regime (log-normal)	18.05	0.3353	0.4648	0.7937	−0.7875
2-regime (Pareto)	33.32	0.5410	0.7851	2.1688	−2.2602

the Gaussian and log-normal models and overestimated by the Pareto distribution. This is somewhat surprising when we recall the results of Case Study 4.2. There the spike severities were very well approximated by a power-law of order $x^{-1.4}$. Here the exponent is well above 4, i.e. it has lighter tails. Yet the spike severities are overestimated. Perhaps the estimation procedure yields a relatively too large scaling parameter.

5. Conclusions. After reviewing the stylized facts of electricity prices and analyzing return distributions we have elaborated on two distinct electricity price models: jump-diffusion and regime-switching. Since both models have been calibrated using "real world" data (i.e. spot prices) we need to include the risk premium before we can use them for pricing options or other derivatives. One way of finding the market price of risk λ is to imply it from option prices (or other derivatives). This technique resembles recovery of the implied volatility in the Black-Scholes model. The procedure consists of finding λ^* such that it minimizes the mean squared error between the market and model option prices. This technique has been used, for instance, by Cartea and Figueroa (2005), Lucia and Schwartz (2002) and Weron (2005a) who calibrated either a constant or a linear λ and applied the method for pricing electricity derivatives.

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